

Solving the discrepancy between the seismic and photospheric solar radius

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ABSTRACT

Two methods are used to observationally determine the solar radius: One is the observation of the intensity profile at the limb, the other one uses f-mode frequencies to derive a 'seismic' solar radius which is then corrected to optical depth unity. The two methods are inconsistent and lead to a difference in the solar radius of ~ 0.3 Mm. Because of the geometrical extension of the solar photosphere and the increased path lengths of tangential rays the Sun appears to be larger to an observer who measures the extent of the solar disk. Based on radiative transfer calculations we show that this discrepancy can be explained by the difference between the height at disk center where $\tau_{5000} = 1$ ($\tau_{\text{Ross}} = 2/3$) and the inflection point of the intensity profile on the limb. We calculate the intensity profile of the limb for the MDI continuum and the continuum at 5000 \AA for two atmosphere structures and compare the position of the inflection points with the radius at $\tau_{5000} = 1$ ($\tau_{\text{Ross}} = 2/3$). The calculated difference between the 'seismic' radius and the inflection point is 0.347 ± 0.06 Mm with respect to $\tau_{5000} = 1$ and 0.333 ± 0.08 Mm with respect to $\tau_{\text{Ross}} = 2/3$. We conclude that the standard solar radius in evolutionary models has to be lowered by 0.333 ± 0.08 Mm and is 695.66 Mm. Furthermore, this correction reconciles inflection point measurements and the 'seismic' radii within the uncertainty.

Subject headings: astrometry — Sun: fundamental parameters — Sun: photosphere — radiative transfer

1. Introduction

The solar radius is a key parameter for understanding the Sun's evolution and many astrophysical applications. Being our closest star, the Sun can be considered as our laboratory, allowing us to determine its radius with high precision. Over the past 25 years various groups were dedicated to determine the solar radius by applying basically two approaches. First, there is the direct measurement of the photospheric radius. In earlier times its visual determination was carried out by an observer. Later, employing CCD-imaging devices,

the visual radius determination was replaced by deriving the inflection point to the intensity profile of the limb. The error of both methods has been investigated in detail by Laclare et al. (1996) who derive a solar radius of $959''.60$, which agrees well with $959''.63 \pm 0''.10$ or 695.99 ± 0.07 Mm (Allen 1976), commonly used for calibrating solar evolutionary models (Brown & Christensen-Dalsgaard 1998; Turck-Chièze & Murdin 2000). Second, Schou et al. (1997); Antia (1998) determine the helioseismic radius by means of f-mode frequencies obtained from the Michelson Doppler Imager (Scherrer et al. 1995, MDI) on board the

ESA/NASA spacecraft SOHO. Due to the high quality of the MDI observations the helioseismic determination is very precise and leads to a radius that is approximately 0.3 Mm smaller than the standard value by Allen (1976). Schou et al. (1997) conclude that the standard solar radius has to be decreased by this value. Finally, Brown & Christensen-Dalsgaard (1998) determine the solar radius from observations obtained with the High Altitude Observatory's Solar Diameter Monitor (Brown et al. 1982, HAO SDM) by applying a model of the data reduction procedure. The authors point out that there are significant uncertainties associated with the currently used radius value which are related to the definition of the solar limb adopted in the radius determinations and the reduction of the measured value to the photosphere. The authors indicate a height of about 0.3 Mm which 'perhaps' explains the radius correction inferred from the f-mode frequencies. In the present paper we determine the difference between the height at $\tau_{5000} = 1$ at disk center and the distance of the inflection point from radiative transfer calculations for two wavelengths and atmosphere structures.

2. Calculations

With the radiative transfer code COSI we calculate the solar limb function for the MDI continuum intensity and the continuum at 5000 Å. COSI is a combination of a model atmosphere code in spherical symmetry, developed by Schmutz et al. (1989) and the spectrum synthesis program SYNSPEC, going back to Hubeny (1981) and further developed by Hubeny & Lanz (1992a,b). The adaption to the solar atmosphere has been published in Haberreiter & Schmutz (2003). The model atmosphere code calculates the NLTE population numbers for a set of specified atomic levels by solving the radiative transport equations simultaneously with the equations for statistical equilibrium. The radiative transfer is solved along rays parallel to the central ray incident on a spherical distribution of the physical atmosphere structure (Mihalas et al. 1975). A spherical geometry allows to calculate geometrically correct the emerging intensity at the limb and line of sights beyond the solar limb.

We base the model calculations on two model

atmosphere structures for the quiet Sun, the LTE structure by (Kurucz 1991, hereafter Kurucz91) and the NLTE structure Model C by Fontenla et al. (2007, hereafter Fontenla07). The radius at disk center where $\tau_{5000} = 1$, is set to the solar radius derived from seismic observations, $R_{\odot} = 695.68$ Mm as given by Schou et al. (1997). The high resolution Ni line profile has been convolved with the five MDI filter functions F0 to F4 used to determine the actual continuum intensity observed by MDI.

3. Results

Fig. 1 panel (a) shows the mean intensity variation for the MDI continuum at the solar limb applying all five MDI filter functions and the continuum at 5000 Å calculated with the LTE and NLTE model atmosphere structures. Distance zero refers to the 'seismic' solar radius $R_{\odot} = 695.68$ Mm, i.e. $\tau_{5000} = 1$ at disk center. For the exact determination of the inflection point we calculate the derivative dI/dx (x being the outward coordinate) of all four limb profiles, shown in Fig. 1 panel (b). The two atmosphere structures agree well in the photosphere and lead to very similar intensity profiles. The derivative of the intensity is always negative and shows a local minimum at the inflection point. The distances of the local minima for all four limb profiles are given in Table 1. The mean value of the distance for the two atmosphere structures for $\lambda = 5000$ Å is 0.347 ± 0.06 Mm, where we used the difference between the two atmosphere structures as criterion to derive the error introduced by the atmosphere structures. This result is close to the value of 0.370 Mm estimated by Wittmann (1974) based on the atmosphere structure by Holweger (1967).

In evolutionary models the radius is typically defined as the point where the temperature equals the effective temperature, i.e. where $\tau_{\text{Ross}} = 2/3$. From our calculations we find that the height where $\tau_{\text{Ross}} = 2/3$ is in the mean 0.014 Mm further out than where $\tau_{5000} = 1$. Thus the correction of an inflection point measurement to the 'seismic' radius is 0.333 Mm. This correction explains the discrepancy between the modeled and observed f-mode frequencies. We conclude that the standard radius used in the evolutionary models has to be corrected by value this value, leading

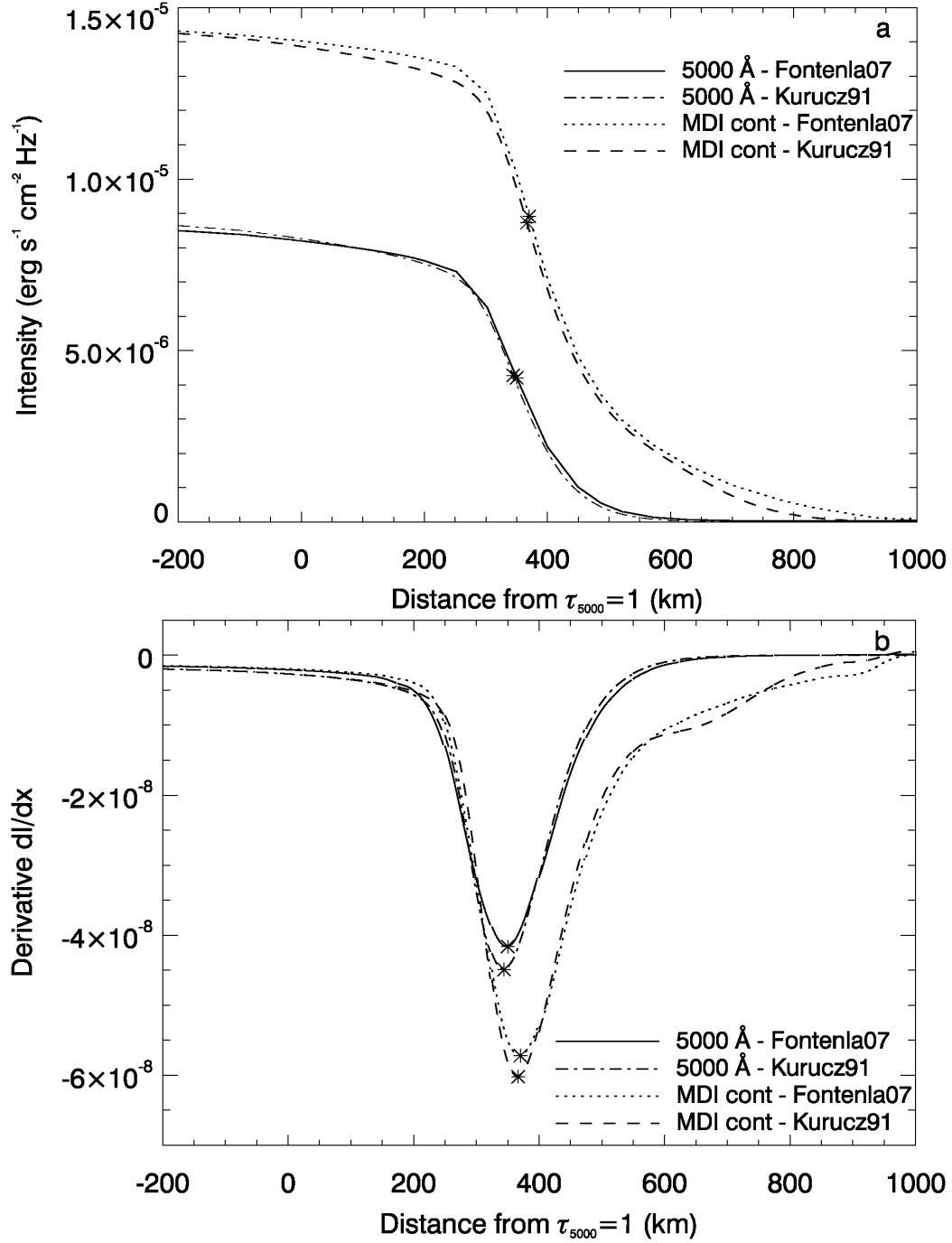


Fig. 1.— Panel (a) gives the intensity profile calculated with the Fontenla07 and Kurucz91 solar atmosphere structures for the 5000 Å continuum (dotted and dashed-dotted) and the MDI continuum (solid and dashed). Panel (b) shows the derivative of the four intensity profiles. The local minimum is the position of the inflection point, which is 365 km for MDI and 347 km for 5000 Å (see Table 1). Note that the distance to $\tau_{\text{Ross}} = 2/3$ is 14 km less.

to 695.66 Mm.

Table 1: Distance z (Mm) between the inflection point and the $\tau_{5000} = 1$ radius ($R_{\odot}=695.68$ Mm) for two different atmosphere structures and wavelengths. The distance to $\tau_{\text{Ross}} = 2/3$ is in the mean 0.014 Mm less.

Atmosph. struct.	$\tau_{5000} = 1$	
	MDI cont.	5000 Å cont.
Kurucz91	0.367	0.344
Fontenla07	0.370	0.350
Δz	0.003	0.006
\bar{z}	0.368	0.347

4. Discussions

For consistency we investigate whether recent inflection point measurements are compatible with the mean of 'seismic' radii relating to $\tau_{\text{Ross}} = 2/3$, i.e. where the temperature equals the effective temperature, by applying this correction term. Table 2 gives the solar radii based on ii) IP measurements, ii) 'seismic' radii. The mean values deviate by ~ 0.24 Mm. In Table 3 we correct the mean radius values of the inflection point observation by 0.333 Mm and compare the result with the mean 'seismic' radius, leading to a difference of 0.09 ± 0.17 Mm. We conclude that the inflection point measurements and the 'seismic' radius can be reconciled by applying the correction of 0.333 Mm.

Our calculations indicate that the position of MDI continuum inflection point is by 0.02 Mm further out than for 5000 Å. The radius value determined by Kuhn et al. (2004) (see Table 2) refers to the inflection point of the intensity profile observed by MDI. The difference of 0.02 Mm cannot explain the somewhat lower radius value by Kuhn et al. (2004).

We emphasize that we do not consider any variation of the solar diameter with time (Emilio et al. 2007; Thuillier et al. 2005; Kuhn et al. 2004; Toulmonde 1997; Parkinson et al. 1980), which however could explain some of the deviation of the radius values. Furthermore, our considerations are with respect to the quiet Sun.

Table 3: Correction of 0.333 Mm applied to the mean of the inflection point measurements. The corrected radius value is compared with the mean of the 'seismic' radii given in Table 2, referring to $\tau_{\text{Ross}} = 2/3$. The result is consistent within the uncertainty.

Radius (Mm)	
\bar{R}_{IP}	695.901 ± 0.098
Correction	0.333 ± 0.008
Corrected \bar{R}_{\odot}	695.568 ± 0.098
\bar{R}_{\odot}	695.658 ± 0.140
ΔR	0.090 ± 0.171

5. Conclusions

From radiative transfer calculations with the spherical code COSI we determine a distance of 0.333 ± 0.008 Mm (0.347 ± 0.06 Mm) between the height where $\tau_{\text{Ross}} = 2/3$ ($\tau_{5000} = 1$) at disk center and the position of the inflection point of the 5000 Å intensity profile at the limb. This correction explains the differences between the f-mode frequencies derived from models calculations and observations. Therefore, we conclude that standard solar radius currently used in evolutionary models has to be corrected by 0.333 Mm and is 695.66 Mm.

Furthermore, the correction of 0.333 Mm reconciles the radius values derived from inflection point measurements and the 'seismic' radius by 0.09 ± 0.17 Mm. Due to the increasing precision of the instruments it becomes necessary that the community agrees on a generally binding definition of the solar radius and that values derived via different techniques are to be corrected accordingly. We propose that the solar radius refers to $R_{\odot} = R(\tau_{\text{Ross}} = 2/3)$. Thus, we suggest that the value of the solar radius determined from inflection point measurements has to be lowered by 0.333 Mm to be comparable with 'seismic' radii. A detailed study of the position of the wavelength and solar cycle dependent inflection point will be crucial for upcoming observations with the SODISM instrument on board PICARD (Thuillier et al. 2006).

This work has been supported by the SNF with

Table 2: Solar radii and their mean values \bar{R}_{\odot} derived by means of i) the standard solar radius and recent inflection point measurements ii) values referring to the 'seismic' radius. As the result given by Wittmann (1997) more than 8σ different from the mean we excluded this value from our considerations.

Authors	R_{\odot} (Mm)	R_{\odot} (arcsec)	Description	\bar{R}_{\odot} (Mm)
<i>i) IP measurements</i>				
Allen (1976)	695.990 ± 0.070	$959''.63 \pm 0''.02$	standard value	
Sofia et al. (1994)	695.917 ± 0.043	$959''.53 \pm 0''.06$	SDS	
Neckel (1995)	695.982 ± 0.022	$959''.62 \pm 0''.03$	McMath ST	
Laclare et al. (1996)	695.830 ± 0.007	$959''.41 \pm 0''.01$	OCA astrolab	
Kuhn et al. (2004)	695.740 ± 0.110	$959''.29 \pm 0''.15$	MDI	
Emilio & Leister (2005)	695.946 ± 0.029	$959''.57 \pm 0''.04$	SP astrolab	695.901 ± 0.098
Wittmann (1997)	696.715 ± 0.029	$960''.63 \pm 0''.04$	Obs. del Teide	
<i>ii) 'seismic' radii</i>				
Schou et al. (1997)	695.680 ± 0.030	$959''.20 \pm 0''.01$	MDI f-modes	
Antia (1998)	$695.787 \pm$	$959''.35 \pm$	GONG f-modes	
Brown & CD (1998)	695.508 ± 0.026	$958''.97 \pm 0''.04$	HAO SDM	695.658 ± 0.140

the Project No. 200020-109420.

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